A Stable and Efficient Admittance Method Via Adjacence Graphs and Recursive Thresholding

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***Abstract*—The generalized admittance method is a rigorous full-wave approach for the analysis of waveguide circuits. Unfortunately, it may present the risk of ill conditioning, especially when very complex structures are analyzed with a considerably high number of modes. In this paper, the concepts of adjacence graph and recursive thresholding are proposed to solve its numerical problems. By applying the proposed strategy, the linear system representing the core of the analysis is partitioned into many independent and well-conditioned subsystems, thus improving the numerical stability of the approach and its efficiency. The attractive features of the proposed approach are its simplicity and immediate implementation. Results are given, referred to a real industrial case, a complex E/H-plane filter, whose analysis could not be performed via a standard admittance method when a very high number of modes were considered. With the present approach, the ill conditioning is avoided and considerable enhancements in computing times is achieved.**

***Index Terms*—Adjacence graphs, generalized admittance method, recursive thresholding, sparse matrices.**

I. Introduction

THE complexity of microwave (MW) waveguiding circuits and components is continually increasing. This is due partly to the technological evolution and partly to the rapid progress in computer-aided design (CAD) and computer machines, which allows the analysis of quite complex structures. As a consequence, the development of numerical methods, efficient and accurate, for the analysis of MW circuits, is becoming crucial. Several approaches have been proposed in the literature for a full-wave rigorous solution of the problem. Among them, the mode-matching (MM) approach is extremely attractive and probably the most used for this class of problems for its high efficiency and accuracy. Thus far, many different formulations of this approach have been presented [1]–[3]: they are all accurate and more or less performed depending on the characteristics of the problem.

More recently, a new formulation has been proposed based on the use of the generalized admittance matrix (GAM) method [4]–[7], which considers a MW circuit as composed of parallel-epipedal elementary cells connected one another. The principle is a "divide and conquer" strategy, in accordance with a common trend in the analysis of very complex circuits, based on a problem segmentation into subdomains [8], [9], possibly analyzed with different methods [10], [11]. This methodology is also open to an efficient and smooth migration toward parallel or distributed environments, which is often the only way to solve very large and complex circuits in an affordable time given the constraints of industrial design [12].

It has been shown [13] that one of the most noticeable advantages of the admittance method is that it allows to use the adjoint network formulation [14] for optimization problems. This results in the generation of a relatively "large" matrix, which contains all the relevant information. An open problem in the use of the GAM formulation in industrial software tools for MW engineering is the existence of risks of numerical ill conditioning in the solution of a large linear system representing the core of the approach. In fact, for computation accuracy purposes, a large number of localized modes are often considered at each discontinuity, thus resulting in the GAM ill conditioning.

Unfortunately, the elimination of higher order modes corresponds to a rather costly (in terms of efficiency) operation since it requires terminating all the considered higher order modes with their characteristic impedance. In this paper, it is shown that, with the use of the adjacence graph (AG) and recursive thresholding (RT), the latter operation can be avoided and the global linear system can be partitioned into independent well-conditioned subsystems. This way, the complexity of the problem is reduced, with a consequent improvement in computing times.

The paper is structured as follows. In Section II, the GAM formulation and its numerical properties are briefly resumed and, in Section III, the AG–RT strategy is proposed to solve its numerical problems. In Section IV, results are given for a real complex industrial case. Finally, conclusions are drawn.

II. GAM Formulation

The method has been described in several publications [4]–[7] and the reader is referred there for details. The GAM formulation is based on the partitioning of a metallic waveguide complex structure into simple volume elements, to be analyzed independently.

The numerical description of the single cell is given in terms of an admittance-type matrix so that the problem of the interconnection among blocks can be handled in merely circuital terms, by imposing to the equivalent voltages and currents of each cell's ports the constraints arising from the overall circuit topology. The analysis of a complex structure is, therefore, reduced to that of the overall equivalent network resulting from the interconnection of its constituent elements [15].

A. GAM of a Resonant Cavity

The single-cell analysis, leading to its Y-matrix characterization, can be performed in a general fashion, by regarding the

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cell, whatever its shape, as a metallic resonator (sourceless inside) having on its surface S an arbitrary number N of apertures S_i, where we suppose an electric field $\mathbf{E}_s$ to be impressed.

The knowledge of the tangential components of $\mathbf{E}_s$ on the apertures, together with that of the tangential component of $\mathbf{E}$ on the rest of S (i.e., on the metallic enclosure, where such a component vanishes), uniquely determines the field inside the cavity. The resulting expression for the magnetic field is

$$\begin{aligned}\underline{\mathbf{H}}(\mathbf{r}) &= j\omega\varepsilon \int_{\bigcup_{i=1}^{N} S_i} \underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}') \cdot \left[\mathbf{n} \times \mathbf{E}_s(\mathbf{r}')\right] d\mathbf{r}' \\ &= j\omega\varepsilon \sum_{i=1}^{N} \int_{S_i} \mathbf{E}_i(\mathbf{r}') \cdot \left[\underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}') \times \mathbf{n}\right] d\mathbf{r}'. \end{aligned} \tag{1}$$

In (1) , $\mathbf{r}$ indicates the observation point, $\mathbf{r}'$ is the source point (i.e., any point of $\bigcup_{i=1}^{N} S_i$), the quantity $\mathbf{n} \times \mathbf{E}_s(\mathbf{r}')$ ($\mathbf{n}$ being the inward normal to S) may be seen as an impressed surface magnetic equivalent current density, $\mathbf{E}_i$ is the electric field impressed on the i-th aperture, and $\underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}')$ is the dyadic magnetic Green's function, a solution of the equation

$$\nabla \times \nabla \times \underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}') - k^2 \underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}') = \underline{\underline{I}}\,\delta(\mathbf{r}-\mathbf{r}') \tag{2}$$

(where $k^2 = \omega^2\mu\varepsilon$, $\underline{\underline{I}}$ is the unit dyadic and $\delta(\mathbf{r}-\mathbf{r}')$ is the Dirac function), with the boundary condition

$$\mathbf{n} \times \nabla \times \underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}') = 0, \qquad \text{on } S$$

meaning the vanishing of the electric-field tangential component on the whole enclosure in presence of a pulse source.

For the sake of simplicity, we rewrite (1) as

$$\mathbf{H}(\mathbf{r}) = \sum_{i=1}^{N} \int_{S_i} \mathbf{E}_i(\mathbf{r}') \cdot \underline{\underline{G}}^{A}(\mathbf{r},\mathbf{r}') d\mathbf{r}' \tag{3}$$

by letting

$$\underline{\underline{G}}^{A}(\mathbf{r},\mathbf{r}') = j\omega\varepsilon \underline{\underline{G}}^{H}(\mathbf{r},\mathbf{r}') \times \mathbf{n}$$

be the *admittance dyadic Green's function*.

The relationship among the electric and magnetic fields at the apertures (respectively, excitation and test quantities in the GAM formulation) can be expressed in terms of an admittance matrix, by expanding the fields at the apertures into a suitable set of ortonormal eigenfunctions as

$$\begin{aligned}\underline{\mathbf{E}}_i &= \sum_{k,l} V_{kl}^{i} \underline{\mathbf{e}}_{kl} \\ \underline{\mathbf{H}}_j &= \sum_{m,n} I_{mn}^{j} \underline{\mathbf{e}}_{mn}, \end{aligned} \tag{4}$$

where the series are to be truncated to the first N_i and N_j terms, respectively, for numerical computing.

In (4), the double indexes kl and mn take into account the bidimensional extension of the apertures, while i and j refer to the aperture being examined. The V_{kl}^{i} and I_{mn}^{j} represent, respectively, the equivalent voltages and currents at the klth and mnth ports.

If we now substitute in the first member of (3) the expression in (4) for $\mathbf{H}_j$ and in its second member the expression in (4) for $\mathbf{E}_i$ and we test the resulting relationship with the general $\mathbf{h}_{mn}(\mathbf{r})$ eigenfunction, we obtain

$$I_{mn}^{j} = \sum_{i=1}^{N} \sum_{k,l} Y_{mn,kl}^{ji} V_{kl}^{i} \tag{5}$$

where

$$Y_{mn,kl}^{ji} = \int_{S_j} \int_{S_i} \mathbf{h}_{mn}(\mathbf{r}) \cdot \underline{\underline{G}}^{A}(\mathbf{r},\mathbf{r}') \cdot \mathbf{e}_{kl}(\mathbf{r}') d\mathbf{r}' d\mathbf{r} \tag{6}$$

defines the GAM of the cavity, which is a square matrix of dimension $\sum_{i=1}^{N} N_i$ (N_i being the number of modes retained to represent the field on the ith aperture).

The $Y_{mn,kl}^{ji}$ element represents the intensity of the mnth equivalent current component induced on the jth aperture (the test one) by the klth equivalent voltage component impressed on the ith aperture (the excitation one).

At this point of the analysis, we can simply regard the cell as an electric network, having a number of ports equal to the number of modes retained in the aperture field expansions. An example of the dyadic Green's function $\underline{\underline{G}}^{A}(\mathbf{r},\mathbf{r}')$ expression for a rectangular resonator can be found in [7] where, for brevity, the reader is referred.

B. Numerical Characteristics of the GAM Formulation

As is quite well known [9], [16], many frequency-domain numerical methods for MW circuit analysis and the GAM approach makes no exception, and have their bottleneck in the solution of a linear system, which, in the GAM case, assumes the form

$$\mathbf{I} = \mathbf{Y}_{\mathbf{G}} \mathbf{V}. \tag{7}$$

$\mathbf{V}$ is the vector of unknown voltages inside the circuit and at its output ports, while $\mathbf{I}$ is the vector of excited currents inside the circuit and over its output ports. The system size can easily reach large dimensions and the system (7) must be solved for each spot frequency. Last but not least, when very large and complex problems are attacked, the condition number of the system can result in a critical value, thus leading the approach to a failure or to large number of iterations when iterative sparse solvers can be used because of the $\mathbf{Y}_{\mathbf{G}}$ matrix sparsity.

Consequently, it can be stated that the current GAM formulation has two main directions of improvements: avoid risks of $\mathbf{Y}_{\mathbf{G}}$ ill conditioning and reduce solution times for system (7)

Both goals are achieved jointly in this paper and the strategy to pursue them is now described. For the sake of clearness, we first describe the proposed AG–RT strategy on simple and academic examples and then discuss the whole methodology by referring to a real industrial case, i.e., the analysis of a complex $\mathbf{E}/\mathbf{H}$-plane filter.

III. System Segmentation Using AG–RT

A. Background

The idea of enhancing the efficiency of the solution of electromagnetic problems by focusing on the solution of linear systems is quite common. A number of techniques have been proposed for many different problems and, among them, a

large class is basically inspired by the principle of reducing the order of the problem, by decomposing the system matrix into suitable fragments, or the solution space into smaller subspaces. Possible examples are the impedance matrix localization (IML) technique [17], the multilevel matrix decomposition algorithm (MLMDA) [18], the fast multipole method (FMM) [19], or techniques based on Krylov subspaces for nonlinear problems [20]. We also have reduction techniques suitable for eigenvalue sparse problems based on Lanczos solvers [21], [22] or derivations [23].

In general, the basic idea is that the whole problem or, equivalently, its system matrix, can generally be decomposed into blocks, with limited reciprocal interactions. This is quite apparent, for instance, in the MLMDA case, where an immediate physical flavor is given to this decomposition by referring to far- and near-field interactions.

The idea proposed in this paper, described in the following section, shares several common issues with the above-mentioned approaches. Nonetheless, we believe it can add a useful contribution for the following reasons.

- It is very simple and general.
- It is easy to implement (not always a consequence of the previous item).
- It shows good speed-ups and improves the system stability

As a consequence, the focus is not on its efficiency and effectiveness with respect to previous techniques, but on its affordability with very little programming efforts, without changing the core of the electromagnetic formulation of the problem.

B. Simple Tutorial Examples

We first consider a very lucky and simple example, represented by the following system matrix:

$$A = \begin{pmatrix} 1 & 0.1 & 0 & 0 \\ 0.1 & 0.4 & 0 & 0.002 \\ 0 & 0 & 0.3 & 0.1 \\ 0 & 0.002 & 0.1 & 0.2 \end{pmatrix}. \tag{8}$$

In this case, it is quite apparent that, if we neglect all the entries smaller than, for instance, 0.005, the linear system is immediately decomposed into two independent systems of size 2, as the *thresholded* matrix is

$$\hat{A} = \begin{pmatrix} 1 & 0.1 & 0 & 0 \\ 0.1 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0.1 \\ 0 & 0 & 0.1 & 0.2 \end{pmatrix}. \tag{9}$$

In this simple case, a single thresholding is enough to identify two independent subproblems. Of course, this is not so typical. Consider, for instance, the following system matrix:

$$A = \begin{pmatrix} x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x \\ 0 & x & x & x & 0 & 0 & 10^{-8} & 0 & 0 \\ 0 & x & x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x & 0 & x & x & 0 & 0 & 10^{-8} & 0 \\ 0 & 0 & 0 & x & x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & x & 0 & 10^{-7} & 0 \\ 0 & 10^{-8} & 0 & 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 10^{-8} & 0 & 10^{-7} & x & x & x \\ x & 0 & 0 & 0 & 0 & 0 & x & x & x \end{pmatrix} \tag{10}$$

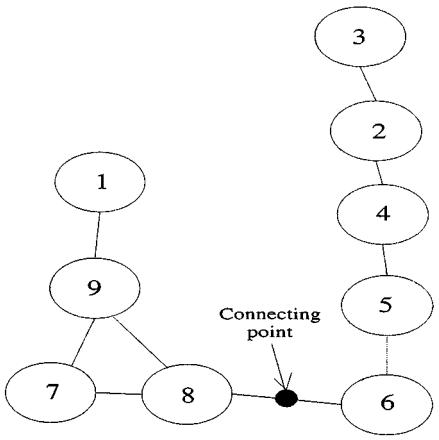

Fig. 1. AG representation of matrix (11). One connecting point is identified. If the corresponding entry in the matrix ($\hat{a}_{68}$ or, equivalently, $\hat{a}_{86}$) is neglected, the graph is partitioned into two independent subgraphs.

where entries generally larger than 10^{-7}, for the sake of simplicity, are represented with x. In this case, if we perform a first thresholding, neglecting values smaller than $v_t = 10^{-8}$, we are not able to identify any independent subproblems, as the resulting system matrix is

$$\hat{A} = \begin{pmatrix} x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x \\ 0 & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & x & x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x & 0 & x & x & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & x & 0 & 10^{-7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & 0 & 10^{-7} & x & x & x \\ x & 0 & 0 & 0 & 0 & 0 & x & x & x \end{pmatrix}. \tag{11}$$

It is now extremely useful to introduce the concept of AG. Each row/column of the matrix is numbered and represented by a node identified with the corresponding number. An arc connects nodes i and j if and only if the entry $\hat{a}_{ij}$ in the matrix is not a zero. The graph representation of matrix (11) is reported in Fig. 1. The nodes in the figure are disposed so that it is apparent that the graph is potentially partitionable into two subgraphs, provided that the entry $\hat{a}_{68}$ (or, equivalently, $\hat{a}_{86}$) is negligible. This entry is defined as a *connecting point*. This is the case, provided that a new thresholding action is performed, with $v_t = 10^{-7}$. Now, we actually have identified two independent subproblems, corresponding to rows/columns 1, 7–9 and 2–6.

The former substantial advantage of AGs is that the identification of *connecting points* is quite straightforward with simple programs. In the simple case of matrix (11), their usefulness is only slightly demonstrated because of the small size of the problem. In real cases, a graph approach is the only viable strategy. The latter advantage is that it can be performed in a very efficient way by using some techniques for graph search and management already developed by the authors and tested on a wide range of numerical techniques for the analysis of MW circuits and antennas [24], [25].

In real cases, it typically happens that many connecting points are identified, each one corresponding to different values of

thresholds. In such cases, an optimum tradeoff between performance enhancements and approximation errors must be pursued (the neglect of a number of matrix entries introduces some errors). In the following section, a discussion on this issue is given referring to a real complex case.

Once the problem has been partitioned into independent subproblems by using a certain threshold v_{t1}, the corresponding submatrices can be determined. For instance, matrix (11) can be partitioned into one submatrix composed of rows and columns 1, 7–9 and the other submatrix 2–6. Afterwards, for each submatrix, the described procedure can be repeated recursively, provided that the new thresholds (eventually different for each submatrix) are larger than v_{t1}. This is why the whole strategy is called AG–RT.

C. Remarks

A key point is the effect of the AG–RT procedure on the condition number of the matrix. While it is quite intuitive that a thresholding might improve the determinant, it is not so obvious its action on the condition number, which is only loosely related with the determinant [26]. An important feature of techniques based on RT is that they improve the stability of the problem, with a reduction of the convergence time when using iterative solvers [27], [28]. This is guaranteed especially when the matrix can be block diagonalized, as in our case, provided that the above-mentioned graph techniques developed by the authors are used [24].

At each iteration of the recursive procedure, the approximation error is increased. Moreover, the percentage reduction of the problem size and the consequent reduction in the numerical complexity of the system solution decreases. Generally, very few recursive steps (fewer than five) are an appropriate choice to achieve a good tradeoff between problem reduction and approximation error.

IV. Results—Application to a Real Case

We consider as a demonstration case a complex circuit, such as the $\mathbf{E}/\mathbf{H}$-plane-step filter of Fig. 2. Its system matrix $\mathbf{Y_G}$ has size 1308 when the functional bases have a cardinality suitable to achieve an appropriate accuracy. The $\mathbf{Y_G}$ matrix is 18% sparse, having 30 386 nonzero entries, and its pattern is shown in Fig. 3. Unfortunately, the condition number is very bad, so that even very sophisticated system solvers are not able to cope with it.

The problem can be solved by using the AG–RT approach. A first thresholding step is performed, with $v_t = 10^{-7}$. The application of the AG identifies 38 connecting points, all corresponding to the used v_t. Consequently, the matrix in Fig. 3 is immediately partitioned into 38 independent subsystems. One of them has dimension 967 (we indicate this submatrix with $\tilde{\mathbf{Y}}_\mathbf{G}$), and all the others are smaller than 90. Apart from the largest subsystem, all the other subsystems have a quite good condition number, with a determinant larger than 10^{-4} and a condition number smaller than 1010^3. Unluckily, the largest subsystem is still ill conditioned and further manipulations are needed in order to solve this relevant part of the problem.

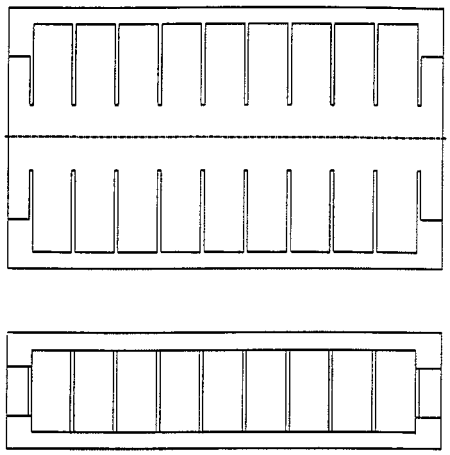

Fig. 2. $\mathbf{H}$- and $\mathbf{E}$-plane view of the filter analyzed by applying the AG–RT strategy.

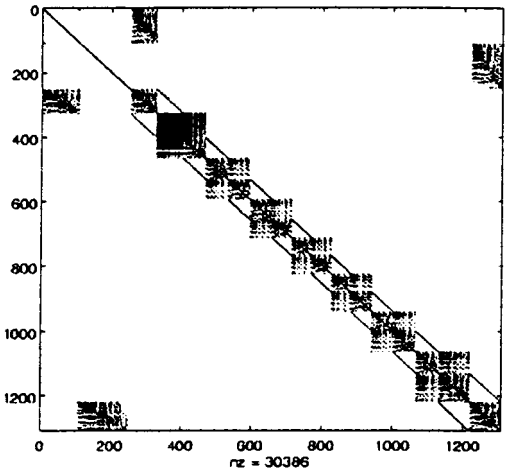

Fig. 3. Zero-nonzero pattern of the GAM of the filter.

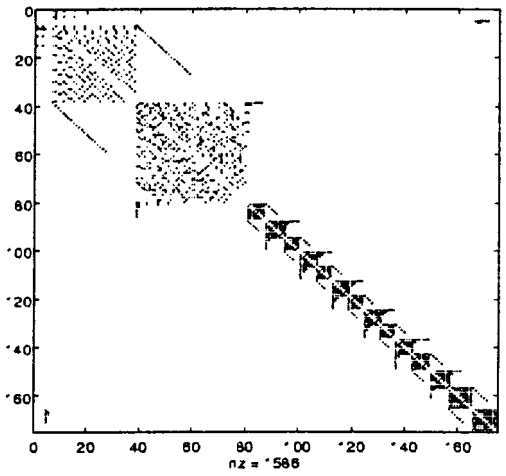

Fig. 4. Largest subsystem attained from the 1308 matrix via the AG–RT matrix partitioning.

Now, a new thresholding and AG step is recursively performed on the $\tilde{\mathbf{Y}}_\mathbf{G}$ submatrix. In this case, a new threshold value $v_t = 5 * 10^{-6}$ is selected, thanks to the identification of many connecting points corresponding to this value and to

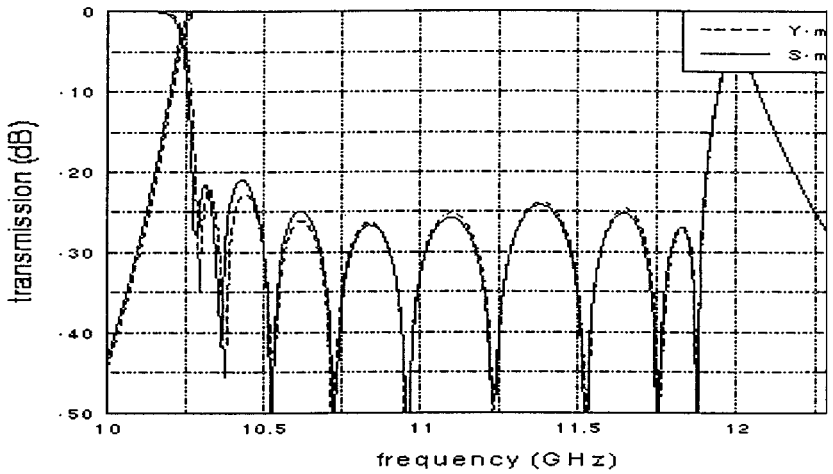

Fig. 5. Scattering parameters for the studied filter, as obtained with two different approaches (the GAM and GSM approaches). The GAM approach has been implemented by following the proposed AG–RT strategy. The dotted curve is for the GAM approach, the continuous curve is for the GSM approach.

the analysis of approximation errors reported in the following subsection. With this choice, the submatrix of size 967 can be partitioned into a large number of submatrices (more than 500). The largest subsystem has size 174 and is shown in Fig. 4. The condition number of nearly all the subsystems is quite good (smaller than 10^5). This finally allows the complete solution of the problem, with a satisfactory degree of accuracy, as reported in Table I and Fig. 5, where the results attained by using the $\mathrm{GAM} + (\mathrm{AG–RT})$ approach are compared with the ones attained with a generalized scattering matrix (GSM) formulation. A two-step AG–RT strategy has been enough to achieve a good tradeoff between accuracy and performance (see Section IV-B). More generally, the strategy is stopped in accordance with a flexible convergence criterion fixed by the user (for instance, once a certain accuracy is reached or a maximum number of iterations). It should also be pointed out that, at each iteration, several connecting points are neglected and the identification of an appropriate v_t is important in order to maximize their number.

A. Approximation Errors

The idea of using a thresholding on matrix entries is common [29], [30] and has also been recently applied for the so-called "wavelet-like" transforms [31]. In the case of a GAM formulation, after several benchmarks on different circuits (rectangular waveguide **E**- and **H**-plane steps, bends, and filters), the results shown in Table I are attained. These results are rather similar to other analyses, performed on planar structures analyzed with the mixed-potential integral equation (MPIE)/method of moments (MoM) [30].

From Table II, we observe that a reasonable tradeoff between accuracy and effectiveness can be achieved by using values of v_t in the range 10^{-6}– 10^{-5}. The identification of a suitable threshold after the matrix normalization is a crucial issue. At the moment it is performed with an *a priori* phenomenological approach, based on the error estimation for a set of possible values. A deterministic procedure is currently under development so that the optimum threshold can be evaluated from the basic properties of the system matrix.

TABLE I
ERROR ON SCATTERING PARAMETERS OBSERVED FOR DIFFERENT CHOICES OF THE THRESHOLD

v_t	Maximum Error on circuit parameters
10^{-7}	0%
10^{-6}	0.2%
10^{-5}	1.1%
10^{-4}	8%

TABLE II
COMPUTING TIMES (IN SECONDS) AS REFERRED TO AN IBM RS6000 390. DATA REFER TO BOTH THE USE OF A SPARSE ITERATIVE SOLVER AND TO THE USE OF THE PROPOSED PARTITIONING STRATEGY IN CONJUNCTION WITH A BANDED SOLVER. BOTH ARE TAKEN FROM SLATEC LIBRARY

Strategy	*Computing Time (s)*
SP	2400
AG-RT	145+21

B. Achieved Speed-Ups

As previously stated, the original problem, of size 1308, has been decomposed into nearly 550 subsystems, ranging from one system of size 174 to many others of smaller size down to size one. All the generated submatrices have a banded structure (see, for example, Fig. 4). In the analyzed case, the application of the AG–RT and the consequent problem decomposition, has allowed the solution of an otherwise ill-conditioned problem.

Nonetheless, the AG–RT strategy is also useful when the conditioning problem is not so severe, as it substantially enhances the time performance of the GAM analysis. Let us consider, for instance, a matrix of the same size, i.e., 1308, with the same sparsity and pattern of the matrix of the circuit in Fig. 2, the same decomposition via AG–RT and a better condition number (for instance, 10^5). The computing time to solve the problem using the AG–RT strategy is substantially reduced with respect to the standard solution of the whole system. We have compared the computing times using a standard sparse solver (SP) to solve the 1308 system versus the strategy proposed here (AG–RT), fragmenting the system and solving all the reduced-size banded subsystems. In Table II, results are given on an IBM RS6000

390. The AG–RT strategy computing time is composed of the time needed to solve all the subsystems (145 s) plus the time needed to fragment and arrange all data (21 s).

As easily noticed, the global speed-up is more than one order of magnitude. The two main factors explaining this improvement are: 1) the subsystems are independent and can be solved concurrently, thus exploiting the multitasking characteristics of nearly all the available computing platforms and 2) the numerical complexity is substantially reduced: the use of direct banded solvers instead of iterative sparse solvers is an advantage, when the matrix size and bandwidth is suitably reduced.

V. Conclusions

In this paper, the introduction of a new strategy based on AGs and RT of the GAM has been proposed. This combined application allows to decompose the linear system matrix into independent submatrices, of smaller dimension, improving the stability of each subsystem and significantly reducing the numerical complexity of the linear system solution.

The implementation of such a strategy is suitable to turn the GAM, which suffers from severe numerical instabilities when using a very high number of modes, into a robust, efficient, and accurate approach for the analysis of waveguide circuits. Moreover, the proposed strategy is general and can, in principle, be applied to a number of different numerical techniques.

As an example, we have proven on a real industrial case that the strategy is a good remedy to the problem of ill conditioning and also allows the enhancement of the numerical efficiency with respect to previous implementation.

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